Exposé plénier

2 The typical size of character sums

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Sums of Dirichlet characters $\sum_{n \le x} \chi(n)$ (where χ is a character modulo some prime r, say) are one of the best studied objects in analytic number theory. Their size is the subject of numerous results and conjectures, such as the Pólya–Vinogradov inequality and the Burgess bound. More generally, one can consider sums $\sum_{n \le x} h(n)\chi(n)$ where h(n) is an interesting twist function, such as the Möbius function. One way to get information about this is to study the power moments $\frac{1}{r-1}\sum_{\chi \mod r} |\sum_{n \le x} h(n)\chi(n)|^{2q}$, which turns out to be quite a subtle question that connects with issues in probability and physics. In this talk I will describe an upper bound for these moments when $0 \le q \le 1$. I will focus mainly on the number theoretic issues arising, and also describe some possible applications of such estimates.