Exposé court

83 *The Li criterion and its variations in the Selberg class*

Mazhouda, Kamel (University of Sousse, Tunisia and UPHF (France))

The Riemann hypothesis (HR), that all non-trivial (non-real) zeros of the Riemann zeta function ζ lie on the critical line $1/2+i\mathbb{R}$, is a conjecture formulated by Riemann in 1859 in the only work he devoted to number theory; (HR) is always open. One of the particular charms of the study of (HR) is the great diversity of its equivalent formulations, which extend to a large class of L functions (the Selberg class, the class of automorphic L functions and the zeta function on function fields). The presentation deals with the study of a relation equivalent to (HR) (the Li criterion and its variations). Furthermore, we reformulate the Li criterion for the Riemann hypothesis for a function F in the Selberg class using some modified Li coefficients defined by

$$\lambda_F(n,a)=\sum_{\rho}\left[1-\left(\frac{\rho-a}{\rho+a-1}\right)^n\right],$$

where the above sum varies over the non-trivial zeros of F and $a\neq 1/2$ is a real number, and we gives an arithmetic and asymptotic formula of $\lambda_F(n,a)$. The main results presented can be found in the paper [1], which is a joint work with Bouchaïb Sodaïgui.

Bibliography

[1] K. Mazhouda and B. Sodaïgui. The Li-Sekatskii coefficients for the Selberg class. *Internat. J. Math.*, 33(12):Paper No. 2250075, 23, 2022. doi:10.1142/S0129167X22500756.