Exposé court

75 *Construction of a normal number in continued fraction and Pisot bases*

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In an integer base b, a normal number is a real number such that every digit blocks of a given length ℓ in its base b expansion appear with the same frequency $\frac{1}{b^\ell}$. For example, Chanpernowne's number $x = 0.12345678910111\ldots$ is known to be normal to base 10. A number is said to be absolutely normal if it is normal to all integer bases, and it is known since their introduction by Borel in 1917 [4] that almost all real number are absolutely normal with respect to Lebesgue's measure. There is however no easy construction of an absolutely normal number, but algorithmic constructions of such numbers exist, such as Turings construction from 1936 [1]. A recent construction is the one of Becher, Heiber and Slaman [2], in which an absolutely normal number is constructed in polynomial time. This construction has been extended to larger sets of bases, Becher and Yuhjtman [3] have constructed a number that is absolutely normal, as well as normal with respect to its continued fraction expansion, and Madritsch, Scheerer and Tichy [5] have constructed a number that is normal to every Pisot base. In this talk, we will present a construction for a number that is normal to every Pisot base, and with respect to its continued fraction expansion.

Bibliography

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