Exposé court

74 *A walk on Legendre paths*

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In this talk, we shall explore what we call "Legendre paths", which are certain paths that encode information about the values of the Legendre symbol. More precisely, the Legendre path modulo p is defined as the polygonal path in the plane, whose vertices are located at the points $(j, S_p(j))$ for $0 \leq j \leq p-1$, where $S_j(p)$ is the normalised sum of the Legendre symbol $\left(\frac{n}{p}\right)$ where n varies from 0 to j. In particular, we will attempt to answer the following questions as we vary over the primes p: how are these paths distributed? how do their peaks behave? and what proportion of the path is above the x-axis? We will see that some of these questions correspond to important and longstanding problems in analytic number theory, including understanding the size of the least quadratic non-residue, as well as the maximum of character sums in the spirit of the Pólya-Vinogradov inequality. Among our results, we prove that as we average over the primes, the Legendre paths converge in law, in the space of continuous functions, to a certain random Fourier series constructed using Rademacher random multiplicative functions. This last result is a joint work with Ayesha Hussain.