Exposé court

72 *Transcendence of infinite products involving binary linear recurrences*

Kurosawa, Takeshi (Tokyo University of Science)

For an algebraic number α, we denote by $\overline{|\alpha|}$ the maximum of the absolute values of its conjugates and by $\mathrm{den}(\alpha)$ the least positive integer such that $\mathrm{den}(\alpha)\,\alpha$ is an algebraic integer, and define $\|\alpha\| = \max\{\overline{|\alpha|}, \mathrm{den}(\alpha)\}$.

Let $\{R_n\}_{n\geq 0}$ be a binary linear recurrence sequence with some conditions. We discuss the transcendence of the infinite product

$$\prod_{k=1}^{\infty}\left(1+\frac{a_k}{R_{r^k}+b_k}\right),$$

where $r \geq 2$ is an integer and a_k and b_k are sequences of algebraic numbers with

$$\log\max(\|a_k\|, \|b_k\|) = o(r^k).$$

We also give new examples of algebraic cases such as

$$\prod_{k=1}^{\infty}\left(1+\frac{2}{\sqrt{5}F_{3^k}-1}\right) = \frac{1+\sqrt{5}}{2},$$

where $\{F_n\}_{n\geq 0}$ is the Fibonacci sequence.

This is joint work with Daniel Duverney (Baggio School for Engineering).