Exposé court

68 *Isogeny classes of typical principally polarized abelian surfaces over $\mathbb{Q}$*

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Faltings proved that the isogeny class of any abelian variety over a fixed number field is finite. This raises a classification question: what can be said about the possible shapes of these isogeny classes?

In the case of elliptic curves over $\mathbb{Q}$, we have a satisfactory answer. Mazur's isogeny theorem lists all the primes ℓ that can appear as the degree of an isogeny over $\mathbb{Q}$, and Kenku showed that every isogeny class has size at most 8. In fact, the possible isogeny graphs (whose vertices are elliptic curves in an isogeny class, and whose edges are irreducible isogenies labeled by degree) can be completely listed.

For higher-dimensional abelian varieties, much less is known on the theoretical side. Nevertheless, one can hope to gain some insight by carrying out explicit computations of isogeny classes. This talk represents a first step in this direction: I will describe an algorithm to compute isogeny classes in the simplest higher-dimensional case, namely principally polarized abelian surfaces over $\mathbb{Q}$ with endomorphism ring equal to $\mathbb{Z}$. The algorithm is practical, and has been employed to constitute a database of more than 1.5 million isogeny classes. This is joint work with Raymond van Bommel, Shiva Chidambaram and Edgar Costa.