Exposé court

64 *Ranks of quadratic twists of Jacobians of generalized Mordell curves*

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Consider a two-parameter family of hyperelliptic curves $C_{q,b} : y^2 = x^q - b^q$ defined over $\mathbb{Q}$, and their Jacobians $J_{q,b}$ where q is an odd prime and without loss of generality b is a non-zero squarefree integer. The curve $C_{q,b}$ is a quadratic twist by b of $C_{q,1}$ (a generalized Mordell curve of degree q). First, we obtain a few upper bounds for the ranks e.g., if $q \equiv 1 \pmod 4$ and any prime divisor of $2b$ not equal to q is a primitive root modulo q then $\operatorname{rank} J_{q,b}(\mathbb{Q}) \leq (q-1)/2$. Then we focus on $q = 5$ and get the best possible bound (by 1) or even the exact value of rank (0). In particular, we found infinitely many b with any number of prime factors such that $\operatorname{rank} J_{5,b}(\mathbb{Q}) = 0$. We deduce as conclusions the complete list (or the bounds for the number) of rational points on $C_{5,b}$ in such cases. Finally, we found for any given q infinitely many non-isomorphic curves $C_{q,b}$ such that $\operatorname{rank} J_{q,b}(\mathbb{Q}) \geq 1$.