Exposé court

53 *The exponential density set*

Grekos, Georges (Université de Saint-Étienne)

Work in common and in progress with Rita Giuliano, Università di Pisa.

Given a nonempty subset A of $\mathbb{N} = \{1, 2, \ldots\}$, its *upper* and *lower exponential densities*, denoted by $\overline{\varepsilon} A$ and $\underline{\varepsilon} A$, are defined, respectively, as the limit superior and the limit inferior of

$$(\log n)^{-1} \log |A \cap [1, n]|$$

when n tends to infinity. The *density set* of A is the set

$$S(A) = \{(\overline{\varepsilon} B, \underline{\varepsilon} B); B \subseteq A\}.$$

We present some properties of $S(A)$.

For the definition of the exponential density and its generalisations, see for instance [1].

The density set corresponding to the *asymptotic* (or *natural*) density was studied for the first time in [2]. Some extensions (concerning the asymptotic density case) are studied in a series of papers; example [3]. It should be noted that in all these cases, the density set is a convex domain of $\mathbb{R}^2$

In the third and final chapter of [2] some other density concepts are considered and a big part of the chapter is devoted to the exponential density set. In this talk I will quickly summarize these results and, going on, I shall present an example which shows that the exponential density set is not necessarily convex. Nevertheless, it has the property of being a starry set with respect to the origin and some other points [*ensemble étoilé par rapport à l'origine et certains autres points*].

Bibliography

[1] R. Giuliano and G. Grekos. On the upper and lower exponential density functions. *Mathematica Slovaca*, 67(5):1105–1128, 2017.

[2] G. Grekos. *Répartition des densités des sous-suites d'une suite donnée.* Thèse de troisième cycle, 1976. Supervised by Jean-Marc Deshouillers.

[3] G. Grekos, L. Mišík, and J. Tóth. Density sets of sets of positive integers. *Journal of Number Theory*, 130:1399–1407, 2010.