Exposé court

5   *Local-global divisibility on algebraic tori*

Alessandrì, Jessica (Università degli Studi dell'Aquila)

The following *local-global divisibility problem* was introduced by R. Dvornicich and U. Zannier in 2001.

Problem 1 (Dvornicich-Zannier, [1])**.** Let k be a number field and $\mathscr{G}$ be a commutative algebraic group. Let q be a fixed positive integer. Assume that a point $P \in \mathscr{G}(k)$ has the following property: for all but finitely many places v of k there exists $D_v \in \mathscr{G}(k_v)$, where k_v is the completion of k at v, such that $P = qD_v$. Can we conclude that there exists $D \in \mathscr{G}(k)$ such that $P = qD$?

In our work we give a complete answer to this problem in algebraic tori for every power of odd primes. Our result is a generalization to any algebraic torus of the Grunwald-Wang Theorem, which provides an answer to Problem 1 in the split case.

The study of the local-global divisibility problem on algebraic tori was started by Dvornicich and Zannier in the same paper [1], who proved that the local-global divisibility for $q = p$ a prime number holds for tori of dimension $r \leq \max(3, 2(p-1))$. Later Illengo in [2] improved the bound on the dimension to $r < 3(p-1)$ and also proved that this bound is sharp.

We prove that the local-global divisibility by any power p^n of an odd prime p holds for algebraic tori over k of dimension $r < p - 1$. We also show that this bound on the dimension is best possible, by providing counterexamples in finite extensions of $\mathbb{Q}$ for every $r \geq p - 1$. Finally, we prove that under certain hypotheses on the number field generated by the coordinates of the p^n–torsion points of T, the local-global divisibility still holds for tori of dimension less than $3(p-1)$.

This is a joint work with Rocco Chirivì and Laura Paladino.

Bibliography

[1] R. Dvornicich and U. Zannier. Local-global divisibility of rational points in some commutative algebraic groups. *Bulletin de la Société Mathématique de France*, 129(3):317–338, 2001.

[2] M. Illengo. Cohomology of integer matrices and local-global divisibility on the torus. *Journal de théorie des nombres de Bordeaux*, 20(2):327–334, 2008.