Exposé court

49 *Second best approximations and the Lagrange spectrum*

Gayfulin, Dmitry (TU Graz)

Given an irrational number α consider its irrationality measure function

$$\psi_\alpha(t) = \min_{1\le q\le t, q\in\mathbb{Z}} \|q\alpha\|.$$

The set of all values of

$$\lambda(\alpha) = \left(\limsup_{t\to\infty} t\psi_\alpha(t)\right)^{-1},$$

where α runs through the set $\mathbb{R}\setminus\mathbb{Q}$ is called the Lagrange spectrum $\mathbb{L}$. Denote by $\mathcal{Q} = \{q_1, q_2, \ldots, q_n, \ldots\}$ the set of denominators of the convergents to α. One can consider another irrationality measure function

$$\psi_\alpha^{[2]}(t) = \min_{1\le q\le t, q\in\mathbb{Z}, q\notin\mathcal{Q}} \|q\alpha\|$$

connected with the properties of so-called second best approximations. Or, in other words, approximations by rational numbers, whose denominators are not the denominators of the convergents to α. Replacing the function ψ_α in the definition of $\mathbb{L}$ by $\psi_\alpha^{[2]}$, one can get a set $\mathbb{L}_2$ which is called the “second” Lagrange spectrum. In my talk I give the complete structure of discrete part of $\mathbb{L}_2$.