

## Exposé court

### 44 **Multiplicative dependence of two integers shifted by a root of unity**

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We prove a result on the multiplicative independence of the numbers  $m - \alpha$ ,  $n - \alpha$ , where  $m > n$  are positive integers and  $\alpha$  is a reciprocal algebraic number with the property that  $\alpha + 1/\alpha$  has at least two real conjugates over  $\mathbb{Q}$  lying in the interval  $(-\infty, 2]$ . As an application, we show that for any positive integers  $m > n$  and  $k \geq 3$  the numbers  $m - \zeta_k$ ,  $n - \zeta_k$ , where  $\zeta_k$  is the primitive  $k$ th root of unity, are multiplicatively independent except when  $(n, k) = (1, 6)$ . This settles a conjecture of Madritsch and Ziegler.