

Exposé court

40 **Quantum j invariant and real multiplication program**

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In my joint work with T. M. Gendron (UNAM, Mexico) we provide a complete solution of Y. Manin's program on the Real Multiplication in the **global function field** context, developing the construction of a **quantum modular invariant** on the moduli space of **quantum tori**. This invariant turns out to be a **multi-valued** function, associating to each point of the moduli space a finite Galois orbit in the *absolute* Hilbert Class Field over the given real quadratic function field K . The norm of these values generates the *relative* Hilbert Class Field over K (the precise analog of the classical Hilbert Class Field in the number field context). We then proceed in constructing a **quantum Drinfeld module** associated to the quantum j , whose torsion points are shown to generate all ray class fields over the relative Hilbert Class Field.

This approach, based on **combining Class Field Theory with Diophantine Approximation** produces therefore encouraging results, which we are now currently working to extend to the number field case by the use of quasi-crystals.

Bibliography

- [1] L. Demangos and T. M. Gendron. Quantum modular invariant and Hilbert class fields of real quadratic global function fields. *Selecta Math. (N.S.)*, 27(1):Paper No. 13, 24, 2021. doi:10.1007/s00029-021-00619-4.
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