Exposé court

32   *Exponential sums with applications in PDEs*

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In 2016 Bourgain applied Gauss sums to construct a counterexample related to a decades-old question in PDEs. The story started in 1980 when Carleson asked about how "smooth" an initial data function must be to imply pointwise convergence for the solution of the linear Schrödinger equation. After progress by many authors, this was resolved by Bourgain, whose counterexample construction proved a necessary condition on the regularity, and Du and Zhang, who proved a sufficient condition. Bourgain's methods were number-theoretic, and this raised a natural question: could number-theoretic properties of other exponential sums have implications for other dispersive PDEs? We develop a flexible new method to construct counterexamples for analogues of Carleson's question. In particular, this applies the Weil bound for exponential sums, a consequence of the truth of the Riemann Hypothesis over finite fields.