Exposé court

22 *Continued fraction expansions of algebraic power series over a finite field*

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Almost nothing is known on the continued fraction expansion of an algebraic real number of degree at least three. The situation is different over the field of power series $\mathbb{F}_p((x^{-1}))$, where p is a prime number. For instance, there are algebraic power series of degree at least three whose sequence of partial quotients have bounded degree. And there are as well algebraic power series of degree at least three which are very well approximable by rational fractions: the analogue of Liouville's theorem is best possible in $\mathbb{F}_p((x^{-1}))$. In a joint work with Han (built on a previous work by Han and Hu), we proved that, for any distinct nonconstant polynomials a, b in $\mathbb{F}_2[x]$, the power series

$$[a; b, b, a, b, a, a, b, \ldots] = a + \frac{1}{b + \frac{1}{b + \cdots}},$$

whose sequence of partial quotients is given by the Thue–Morse sequence (which is a 2-automatic sequence), is algebraic of degree 4 over $\mathbb{F}_2(x)$. Very recently, this has been extended to several families of 2-automatic sequences in a remarkable paper by Hu. In this talk, we give a complete description of the continued fraction expansion of the algebraic power series $(1 + x^{-1})^{j/d}$ in $\mathbb{F}_p((x^{-1}))$, where j, d are coprime integers with $d \geq 3$, $1 \leq j < d/2$, and $\gcd(p, jd) = 1$ (joint work with Guo-Niu Han).