Exposé court

19 *Extreme values of Birkhoff sums and quantum modular forms*

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The notion of a quantum modular form was introduced in a seminal paper of Zagier [3] in 2010 to describe the rich arithmetic structure of certain quantum knot invariants coming from algebraic topology. In this talk we demonstrate that quantum modular behavior also emerges in the ergodic theory of circle rotations. In particular, we consider the extreme values as well as exponential moments of the classical Birkhoff sum $\sum_{n=1}^{N}(\{nr\}-1/2)$, $0 \le N < \mathrm{denom}(r)$ as functions of $r \in \mathbb{Q}$, and establish remarkable transformation properties with respect to the Gauss map that reveals an interesting self-similar structure. Transferring this framework to the irrational setting, as an application we find the limit distribution (after suitable centering and scaling) of $\max_{1\le N\le M}\sum_{n=1}^{N} f(n\alpha)$ and $\min_{1\le N\le M}\sum_{n=1}^{N} f(n\alpha)$ with $f(x)=\{x\}-1/2$ and a randomly chosen real $\alpha \in [0,1]$. The same limit law holds with f being the indicator of a rational interval extended with period 1. The talk is based on the papers [1, 2].

Bibliography

[1] B. Borda. Equidistribution of continued fraction convergents in $\mathrm{SL}(2,\mathbb{Z}_m)$ with an application to local discrepancy. arXiv:2303.08504.

[2] B. Borda. Limit laws of maximal Birkhoff sums for circle rotations via quantum modular forms. To appear in Int. Math. Res. Not. IMRN. arXiv:2303.07796.

[3] D. Zagier. Quantum modular forms. In *Quanta of maths*, volume 11 of *Clay Math. Proc.*, pages 659–675. Amer. Math. Soc., Providence, RI, 2010.