

Exposé court

150 Equal-Sum-Product Problem

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In this talk, we present the results related to a problem posed by Andrzej Schinzel [1], [2, p. 261–262]. Does the number $N_1(n)$ of integer solutions of the equation

$$x_1 + x_2 + \dots + x_n = x_1 x_2 \cdot \dots \cdot x_n, \quad x_1 \geq x_2 \geq \dots \geq x_n \geq 1$$

tend to infinity with n ? Let a be a positive integer. We give a lower bound on the number of integer solutions, $N_a(n)$, to the equation

$$x_1 + x_2 + \dots + x_n = ax_1 x_2 \cdot \dots \cdot x_n, \quad x_1 \geq x_2 \geq \dots \geq x_n \geq 1.$$

We show that if $N_2(n) = 1$, then the number $2n - 3$ is prime. The average behavior of $N_2(n)$ is studied. We prove that the set $\{n : N_2(n) \leq k, n \geq 2\}$ has zero natural density.

Bibliography

- [1] A. Schinzel. Sur une propriété du nombre de diviseurs. *Publ. Math. Debrecen*, 3:261–262 (1955), 1954.
- [2] A. Schinzel. *Andrzej Schinzel selecta. Vol. II*. Heritage of European Mathematics. European Mathematical Society (EMS), Zürich, 2007. Elementary, analytic and geometric number theory. Edited by Henryk Iwaniec, Władysław Narkiewicz and Jerzy Urbanowicz.