Exposé court

142 *Hooley's function along friable integers*

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Introduced by Erdős in 1974, *Hooley's Delta function* is defined as

$$\Delta(n) := \sup_{v\in\mathbb{R}} \sum_{\substack{d|n \\ v\leq \log d < v+1}} 1 \quad (n \in \mathbb{N}^*).$$

Hooley's function is thus an arithmetic function which measures the logarithmic concentration of the set of divisors of an integer. It is known that there exists $c_0 > 0$ such that

$$\log_2 x \ll \frac{1}{x}\sum_{n\leq x} \Delta(n) \ll \exp(c_0\sqrt{\log_2 x \log_3 x}) \qquad (x \geq 16).$$

The lower bound was proved by Maier and Tenenbaum in 1982 and the upper bound by Tenenbaum in 1985.
We obtain lower and upper bounds for the average order of Δ along friable integers, i.e. integers without large prime factors.
To get the lower bound we evaluate the quantity $\sum_{\substack{n\leq x \\ P^+(n)\leq y}} \Delta(n)$ over three non-disjoint domains, using results from saddle point approximations, probabilistic arguments and a parametrized version of a theorem by Tenenbaum and Wu. As for the upper bound, we extend a method developped by Tenenbaum in 1985 of the average value of Δ along friable integers.