Exposé court

139 *Construction of polynomials with prescribed divisibility conditions on the critical orbit*

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Let $f_{d,c}(x) = x^d + c \in \mathbb{Q}[x]$, $d \geq 2$. We write $f_{d,c}^n$ for $\underbrace{f_{d,c} \circ f_{d,c} \circ \cdots \circ f_{d,c}}_{\text{n times}}$. The critical orbit of $f_{d,c}(x)$ is the set $\mathcal{O}_{f_{d,c}}(0) := \{f_{d,c}^n(0) : n \geq 0\}$.

For a sequence $\{a_n : n \geq 0\}$, a primitive prime divisor for a_k is a prime dividing a_k but not a_n for any $1 \leq n < k$. A result of H. Krieger asserts that if the critical orbit $\mathcal{O}_{f_{d,c}}(0)$ is infinite, then each element in $\mathcal{O}_{f_{d,c}}(0)$ has at least one primitive prime divisor except possibly for 23 elements. In addition, under certain conditions, R. Jones proved that the density of primitive prime divisors appearing in any orbit of $f_{d,c}(x)$ is always 0.

In this talk, I'll discuss joint work with Mohammad Sadek, in which we display an upper bound on the count of primitive prime divisors of a fixed iteration $f_{d,c}^n(0)$. Further, we show that there is no uniform upper bound on the count of primitive prime divisors of $f_{d,c}^n(0)$ that does not depend on c. In particular, given $N > 0$, there is $c \in \mathbb{Q}$ such that $f_{d,c}^n(0)$ has at least N primitive prime divisors. This, along with some previous results, allows for the construction of polynomials of the form $f_{d,c}(x)$ whose n-th iterates possess maximal Galois Groups.