Exposé court

137 *Curves with few bad primes over cyclotomic $\mathbb{Z}_\ell$-extensions*

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Let K be a number field, and S a finite set of non-archimedean places of K, and write $\mathcal{O}_S^\times$ for the group of S-units of K. A famous theorem of Siegel asserts that the S-unit equation $\varepsilon + \delta = 1$, with $\varepsilon, \delta \in \mathcal{O}_S^\times$, has only finitely many solutions. A famous theorem of Shafarevich asserts that there are only finitely many isomorphism classes of elliptic curves over K with good reduction outside S. Now let ℓ be a prime, and instead of a number field, let $K = \mathbb{Q}_{\infty,\ell}$ which denotes the $\mathbb{Z}_\ell$-cyclotomic extension of $\mathbb{Q}$. We show that the S-unit equation $\varepsilon + \delta = 1$, with $\varepsilon, \delta \in \mathcal{O}_S^\times$, has infinitely many solutions for $\ell \in \{2, 3, 5, 7\}$, where S consists only of the totally ramified prime above ℓ. Moreover, for every prime ℓ, we construct infinitely many elliptic or hyperelliptic curves defined over K with good reduction away from 2 and ℓ. For certain primes ℓ we show that the Jacobians of these curves in fact belong to infinitely many distinct isogeny classes. This talk is based on joint work with Samir Siksek.