Exposé court

136 Sums of arithmetic functions running on factorials

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Given an arithmetic function $f : \mathbb{N} \to \mathbb{R}$, it is customary to investigate the behavior of the corresponding sum $\sum_{n \le N} f(n)$ for large N. Here, for various classical arithmetic functions f, including the number of distinct prime factors function $\omega(n)$, Euler totient's function $\phi(n)$, the number of divisors function d(n), the sum of the divisors function $\sigma(n)$, as well as the middle divisors functions $\rho_1(n)$ and $\rho_2(n)$, we investigate the behavior of f(n!) and their corresponding sums $\sum_{n \le N} f(n!)$. Finally, if λ stands for the Liouville function, according to the Chowla conjecture, $\sum_{n \le N} \lambda(n)\lambda(n+1) = o(N)$ as $N \to \infty$; here, we show that the analogue of the Chowla conjecture for factorial arguments is true as we prove that, as $N \to \infty$, we have $\sum_{n \le N} \lambda(n!)\lambda((n+1)!) = o(N)$.