Exposé court

120 *Hankel determinants associated with weighted binary sum of digits*

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For a number sequence $(a_n)_{n\in\mathbb{N}}$ its Hankel determinant of order n is defined by

$$H(n) = \det[a_{i+j}]_{0\le i,j<n}.$$

Recently, there has been a lot of interest in Hankel determinants of automatic sequences, such as the Thue–Morse, paperfolding, and related sequences. In particular, results concerning non-vanishing of Hankel determinants allowed to compute irrationality exponents for real numbers with digits given by the terms of these sequences.

In the talk we will focus on Hankel determinants of a closely related family of sequences $(s_{\mathbf{w}}(n))_{n\in\mathbb{N}}$, describing weighted binary sums of digits of $n\in\mathbb{N}$, where $\mathbf{w} = (w_j)_{j\in\mathbb{N}}$ is a sequence of weights. We will provide an explicit formula for the Hankel determinants for $a_n = s_{\mathbf{w}}(n+1) - s_{\mathbf{w}}(n)$, which generalizes a result of Fokkink, Kraaikamp and Shallit for the period-doubling sequence. In the case $a_n = s_{\mathbf{w}}(n)$ we will give a recurrence relation satisfied by $H(n)$, which partially answers a question by Allouche and Shallit concerning usual binary sum of digits. For the weights $w_j = t^j$ and we will give further results concerning vanishing and non-vanishing of infinitely many Hankel determinants, as well as their divisibility in the case $t\in\mathbb{Z}$.

Joint work with Maciej Ulas (Jagiellonian University).