# Exposé court

## 118 On the spaces of spherical polynomials and generalized theta-series for quadratic forms of any number of variables

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Let

$$Q(X) = Q(x_1, x_2, \dots, x_r) = \sum_{1\le i\le j\le r} b_{ij} x_i x_j$$

be an integer positive definite quadratic form of $r$ variables and let $A = (a_{ij})$ be the symmetric $r \times r$ matrix of the quadratic form $Q(X)$, where $a_{ii} = 2b_{ii}$ and $a_{ij} = a_{ji} = b_{ij}$, for $i < j$ and $a^*_{ij}$ is the element of the inverse matrix $A^{-1}$.

A homogeneous polynomial $P(X) = P(x_1, \cdots, x_r)$ of degree $\nu$ with complex coefficients, satisfying the condition

$$\sum_{1\le i,j\le r} a^*_{ij}\left(\frac{\partial^2 P}{\partial x_i \partial x_j}\right) = 0$$

is called a spherical polynomial of order $\nu$ with respect to $Q(X)$. Let $P(\nu, Q)$ denote the vector space of spherical polynomials $P(X)$ of even order $\nu$ with respect to $Q(X)$.

Let

$$\vartheta(\tau, P, Q) = \sum_{n\in\mathbb{Z}^r} P(n) z^{Q(n)}, \qquad z = e^{2\pi i\tau}, \qquad \tau \in \mathbb{C}, \qquad \operatorname{Im}\tau > 0$$

be the corresponding generalized $r$-fold theta-series and $T(\nu, Q)$ denote the vector space of generalized multiple theta-series, i.e.,

$$T(\nu, Q) = \{\vartheta(\tau, P, Q) : P \in \mathscr{P}(\nu, Q)\}.$$

Gooding [1] calculated the dimension of the vector space $T(\nu, Q)$ for reduced binary quadratic forms $Q$. In [2] the upper bounds for the dimension of the space $T(\nu, Q)$ for some quadratic forms of $r$ variables are established.

In this talk, some positive diagonal and non-diagonal quadratic forms of any number of variables are considered; the spaces of spherical polynomials and corresponding generalized theta series are studied; and finally the upper bounds of the dimensions (in some cases, the dimensions) of these spaces are obtained.

### *Bibliography*

[1] F. Gooding, Jr. Modular forms arising from spherical polynomials and positive definite quadratic forms. *J. Number Theory*, 9(1):36–47, 1977. doi:10.1016/0022-314X(77)90047-6.

[2] K. Shavgulidze. On the space of generalized theta-series for certain quadratic forms in any number of variables. *Math. Slovaca*, 69(1):87–98, 2019. doi:10.1515/ms-2017-0205.