Exposé court

116 Stability of certain higher degree polynomials

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Let K be a field and let $f(z) \in K[z]$ be a polynomial of degree d. Then, for $n \in \mathbb{N} \cup \{0\}$, the n-th iterate of $f(z)$ is defined inductively as follows

$$f^0(z) = z, f^n(z) = f(f^{n-1}(z)).$$

A polynomial $f(z) \in K[z]$ is *stable* over K if $f^n(z)$ is irreducible over K for each $n \in \mathbb{N}$. If the number of irreducible factors of all the iterates of $f(z)$ is bounded above by a constant, then we say that $f(z)$ is *eventually stable*. An important question in the field of arithmetic dynamics is to study the stability or eventual stability of polynomials over a field. In this talk, we discuss the stability of $f(z) = z^d + \frac{1}{c}$ for $d \geq 3$, $c \in \mathbb{Z} \setminus \{0\}$. We show that for an infinite family of $d \geq 3$, the irreducibility of $f(z)$ implies stability of $f(z)$; for the remaining values of d, explicit-abc conjecture implies that $f(z)$ is stable whenever it is irreducible. Moreover, for $d = 3$, if $f(z)$ is reducible, then we show that the number of irreducible factors of $f^n(z)$ is exactly 2, for each $n \in \mathbb{N}$ and for $|c| \leq 10^{12}$. This is a joint work with Prof. Ritumoni Sarma and Prof. Shanta Laishram.