# Exposé court

## 112 *Multiplicative complements*

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The set of nonnegative integers is denoted by $\mathbb{N}$. The counting function of a set $A \subseteq \mathbb{N}$ is defined as $A(x) = |A \cap \{0, 1, \ldots, x\}|$ for every $x \in \mathbb{N}$. Let $A, B \subseteq \mathbb{N}$. The sets $A$ and $B$ are said to be additive complements if every nonnegative integers $n$ can be written as $n = a+b$, $a \in A$, $b \in B$. Clearly, if $A, B \subseteq \mathbb{N}$ are additive complements, then $A(x)B(x) \geq x+1$ for every $x \in \mathbb{N}$, therefore $\liminf_{x\to\infty} \frac{A(x)B(x)}{x} \geq 1$. In 1964, answering a question of Hanani, Danzer proved that this bound is sharp, that is there exists infinite additive complements $A, B \subseteq \mathbb{N}$ such that $\lim_{x\to\infty} \frac{A(x)B(x)}{x} = 1$.

Similarly, the sets $A$ and $B$ are said to be multiplicative complements if every nonnegative integers $n$ can be written as $n = ab$, $a \in A$, $b \in B$. We show that, in contrast to the additive complements, $\lim_{x\to\infty} \frac{A(x)B(x)}{x} = \infty$ for every infinite multiplicative complements $A$ and $B$. In this talk we present some further tight density bounds on multiplicative complements.

This is joint work with Anett Kocsis, Dávid Matolcsi and György Tőtős.