Exposé court

110 *Topological properties and algebraic independence of sets of prime-representing constants*

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Let $\lfloor x \rfloor$ us denote the integer part of $x \in \mathbb{R}$. Let $(c_k)_{k=1}^{\infty}$ be a sequence of positive integers. Assume that $(c_k)_{k=1}^{\infty}$ satisfies certain suitable conditions. We investigate the set of $A > 1$ such that $\lfloor A^{c_1 \cdots c_k} \rfloor$ is always a prime number for every positive integer k. Let $\mathscr{W}(c_k)$ be this set. Mills was the first to propose such a constant A. He showed that there exists a constant $A > 1$ such that $\lfloor A^{3^k} \rfloor$ is always a prime number. Therefore, $\mathscr{W}(3)$ is non-empty. The minimum of $\mathscr{W}(3)$ is called Mills' constant. It is still open to determine whether Mills' constant is rational or irrational. Interestingly, Alkauskas and Dubickas constructed a transcendental number in $\mathscr{W}(c_k)$ if $\limsup_{k\to\infty} c_k = \infty$.

The first goal of this talk is to determine the topological structure of $\mathscr{W}(c_k)$. Under suitable conditions on $(c_k)_{k=1}^{\infty}$, we reveal that $\mathscr{W}(c_k) \cap [0, a]$ is homeomorphic to the Cantor middle third set for some real a. The second goal is to propose an algebraically independent subset of $\mathscr{W}(c_k)$ if c_k is rapidly increasing. As a corollary, we disclose that the minimum of $\mathscr{W}(k)$ is transcendental. In addition, we apply the main result to $\mathscr{W}(c_k)$ with $c_1 \cdots c_k = 3^{k!}$. As a consequence, we give an algebraically independent and countably infinite subset of $\mathscr{W}(c_k)$. This research is joint work with Wataru Takeda (Tokyo University of Science).