Exposé court

104 *Monogenity of parametric families of number fields*

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Let $\mathbb{Q} \leq K$ be a field extension of degree n and let $\mathcal{O}_K$ be the ring of integers of K. We say that K is monogenic over $\mathbb{Q}$, if $\mathcal{O}_K$ is mono-generated as a ring over $\mathbb{Z}$, i.e. $\mathcal{O}_K = \mathbb{Z}[\alpha]$ for some $\alpha \in \mathcal{O}_K$. In this case $(1, \alpha, \alpha^2, \ldots, \alpha^{n-1})$ is an integral basis of K and consequently, the index $[\mathcal{O}_K : \mathbb{Z}[\alpha]]$ of α is one. It is a classical problem in algebraic number theory to decide if a number field is monogenic or not.

The first example of a non-monogenic number field was given by Dedekind. His example is based on the fact that if a prime $p \in \mathbb{Z}$ does not divide the index of α, then the prime ideal decomposition of $p\mathcal{O}_K$ is in one-to-one correspondence with the modulo p factorization of the minimal polynomial of α over $\mathbb{Q}$. He proved that one can deal with the monogenity of a number field through the prime ramification if and only if the field index is not 1. Unfortunately, this approach is not complete in the sense that there are non-monogenic number fields with field index 1.

The problem of finding all of the generators of a power integral basis in the number field is equivalent to the problem of solving the index-form equation. It is a Diophantine equation of $n-1$ variables and of degree $\binom{n}{2}$, so it is very complicated to solve in general. However, this approach can be successful even in the case when the field index is 1.

In this talk I mention some classical results and methods concerning the monogenity of infinite parametric families of number fields and some new directions that has been in the scope of the most recent papers.