Titre: " Souvenirs...souvenirs..." par Michel PIERRE, École Normale Supérieure de Rennes et Institut de Recherche Mathématique de Rennes (IRMAR).

Quelques mots-clefs pour cet exposé orienté "Souvenirs": formage électromagnétique, jet de métal liquide, problèmes inverses, bulles en lévitation, calcul de formes optimales, BFGS, Newton, Numath, Vittel ...

Existence, uniqueness and numerical simulation of a renormalized periodic solution to a nonlinear parabolic equation with variable exponent and L_1 data.

Nour Eddine Alaa
Laboratory LAMAI, Faculty of Science and Technology, Cadi Ayyad University, Marrakech, Morroco
E-mail: n.alaa@uca.ac.ma

Abstract: Partial differential equations with variable exponents have been extensively studied in recent years. It is an important task which reveals many mathematical problems and attributes in several models. In this presentation, we consider a nonlinear p(x)-Laplacian parabolic equation with L_1 data and periodic condition in the time, for which we prove the existence and uniqueness of a periodic renormalized solution by using new techniques. Finally we present a numerical algorithm based on artificial neural network to simulate the periodic solution of our problem.
Keywords: p(x)−Laplacian, Parabolic Problems, Periodic solution, Variable Exponent, Renormalized Solutions, L_1 Data, Deep Neural Networks

Body-fitted topology optimization of 2D and 3D fluid-to-fluid heat exchangers

G. Allaire,
CMAP, Ecole Polytechnique

This talk is concerned with a topology optimization approach for the design of fluid-to-fluid heat exchangers which rests on an explicit meshed discretization of the solid and fluid phases, at every iteration of the optimization process.
The considered physical situations involve a weak coupling between the Navier--Stokes equations for the velocity and the pressure in the fluid, and the convection--diffusion equation for the temperature field.
The proposed framework combines several recent techniques from the field of shape and topology optimization, and notably a level-set based mesh evolution algorithm for tracking shapes and their deformations, an efficient optimization algorithm for constrained shape optimization problems, and a numerical method to handle a wide variety of geometric constraints such as thickness constraints and non-penetration constraints.
Our strategy is applied to the optimization of various types of heat exchangers.
A first example is a simplified 2D cross-flow model where the optimized boundary is the section of the hot fluid phase flowing in the transverse direction, which is naturally composed of multiple holes. A minimum thickness constraint is imposed on the cross-section so as to account for manufacturing and maximum pressure drop constraints.
A second example is the design of 2D and 3D heat exchangers composed of two types of fluid channels (hot and cold), which are separated by a solid body.
A non-mixing constraint between the fluid components containing the hot and cold phases is
enforced by prescribing a minimum distance between them.
This is a joint work with C. Dapogny, F. Feppon and P. Jolivet.

Shape and topology optimal design problems in electromagnetic casting

Alfredo Canelas
UDELAR, Université de la République, Montevideo, Uruguay

In this presentation the recent numerical techniques proposed to solve the forward and inverse problems concerning the electromagnetic casting and electromagnetic levitation techniques of the metallurgical industry are reviewed. In addition, a new topology optimization method to solve the inverse axisymmetric electromagnetic levitation problem is presented. The proposed method is based on an exact second-order topological expansion of a Kohn–Vogelius-like functional specially devised for this problem. Through some examples it is shown that the algorithm can find suitable solutions efficiently. This new method completes the set of efficient methods available to solve the inverse electromagnetic casting and the inverse axisymmetric electromagnetic levitation problems.

Interpolation and approximation on the Cubed Sphere grid.

Jean-Pierre Croisille
IECL, Metz

Interpolation and approximation on the sphere
are important and old topics in harmonic and numerical analysis.
They are essential for modelling in many domains such as
numerical climatology, quantum chemistry, neutronic, data analysis on the sphere, etc.
In this talk, we will present several properties and numerical results obtained recently
on the interpolation and approximation problems with spherical harmonics on the
Cubed Sphere.
Applications to spherical quadrature rules will be mentioned.

This is a joined work with J.-B. Bellet and M. Brachet.

Theoretical and numerical analysis of parabolic quasi-linear partial differential equations

Nahed NACEUR
ISTP, Saint-Etienne

The aim of this work is to compute a numerical positive solution of a quasi-linear elliptic or parabolic PDE's equation in two dimensions. We consider quadratic growth of the non-linearity on the gradient term.
Based on the theoretical analysis, we present a numerical method to compute a super-solution of this problem by using the Newton's method and domain decomposition technics.
Then, starting by this super-solution, we construct a sequence of solutions of regularized problems that converges to the numerical positive solution of our equation.
After, we give some numerical examples attesting the performance of the presented method.

A history of semi-Lagrangian methods in plasma physics.

Eric Sonnendrucker,
Max Planck Institute, Garching, Allemagne

Abstract: After the original Cheng and Knorr paper on a grid based method for solving the Vlasov-Poisson equation, we generalized the method for more general Vlasov type equations including Vlasov-Maxwell and gyrokinetic equations. Since then many physics codes have been developed based on this method, including the gyrokinetic code GYSELA used for magnetic fusion simulations. Also on the mathematical side many papers have been published to improve the method in several ways. The talk will present an overview of the method and the main developments that have followed since our 1999 paper

Fractional Elliptic Systems with Gradient Source Terms : Numerical Studies and Simulations

Maha DAOUD

Abstract.

The fractional Laplacians $(-\Delta)^s$ of order $0 < s < 1$ constitute the simplest models of nonlocal elliptic operators (see [1] and the references therein). In this talk, we focus on a class of fractional elliptic systems with gradient source terms (see [2]) of the form

$$(S) \qquad \begin{cases} (-\Delta)^s u(\mathbf{x}) &= \|\nabla v(\mathbf{x})\|^q + \lambda f(\mathbf{x}), \quad \mathbf{x} \in \Omega, \\ (-\Delta)^s v(\mathbf{x}) &= \|\nabla u(\mathbf{x})\|^p + \mu g(\mathbf{x}), \quad \mathbf{x} \in \Omega, \\ u(\mathbf{x}) &= v(\mathbf{x}) = 0, \qquad\qquad\quad \mathbf{x} \in \mathbb{R}^N \setminus \Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded regular domain, $N > 2s$ with $\frac{1}{2} < s < 1$, $p, q > 1$, $\lambda, \mu > 0$, and f and g are measurable nonnegative functions. This class of systems arises in a variety of applications such as Fluid dynamics and Engineering (see, for instance, [3]).

In the first part of this presentation, we briefly present some existence and nonexistence results under natural conditions on the data λ, μ, f and g. Moreover, we discuss many open questions.
In the second part, we expose some numerical studies of some open questions. In fact, our simulations confirm the theoretical results and give rise to interesting conjectures. Furthermore, we numerically illustrate the relation between the fractional systems and the classical ones treated in [4].

References

[1] **M. Daoud** ; E.-H. Laamri, *Fractional Laplacians : a short survey.* Discrete and Continuous Dynamical Systems - Series S, 15(1) (2022), 95-116.

[2] S. Atmani ; K. Biroud ; **M. Daoud** ; E.-H. Laamri, *On some nonlocal elliptic systems with gradient source terms.* To appear in Acta Applicandae Mathematicae.

[3] S. Clain, J. Rappaz, M. Swierkosz, R. Touzani, *Numeical modeling of induction heating for two-dimensional geometries.* Mathematical Models and Methods in Applied Sciences 3(6) (1993), 805-822.

[4] B. Abdellaoui, A. Attar ; E.-H. Laamri, *On the existence of positive solutions to semilinear elliptic systems involving gradient term.* Applicable Analysis 98(7) (2019), 1289-1306.

Global existence for a class of reaction-diffusion systems : a numerical study

Rajae MALEK
Moulay Ismail University, Faculty of Sciences, Meknes - Morocco

Ecology with Lotka-Volterra systems, chemistry with reaction-rate equations, multi-species diffusion of molecules and many other scientific fields lead to reaction-diffusion systems characterized by different diffusion coefficients and satisfying the two natural following properties :
— positivity of the solutions is preserved for all time ;
— the total mass of the components is controlled for all time.

In this presentation, we focus on reaction-diffusion systems modeling reversible chemical reactions. Such systems are of the form

$$(RDS)\begin{cases} 1 \leq i \leq m & & & \\ \partial_t u_i - d_i \Delta u_i & = & f_i(u_1, u_2, \cdots, u_m) & \text{in} \quad (0,T)\times\Omega \\ \partial_\nu u_i(t, \boldsymbol{x}) & = & 0 & \text{on} \quad (0,T)\times\partial\Omega \\ u_i(0,\boldsymbol{x}) & = & u_{0,i}(\boldsymbol{x}) \geq 0 & \text{in} \quad \Omega \end{cases}$$

where for all $i \in \{1, \cdots, m\}$, $d_i > 0$ and $f_i(u_1, \cdots, u_m) = (p_i - q_i)\left(\prod_{j=1}^{m} u_j^{q_j} - \prod_{j=1}^{m} u_j^{p_j}\right)$.

In this talk, we will first recall the known global existence results and the open questions. Then, we will present a numerical study of some open questions. As we will see, our simulations confirm the known theoretical results and give rise to interesting conjecture.

This is a joint work with El Haj Laamri (Université de Lorraine) and Chérif Ziti (Université Moulay Ismail).

Références

[1] Rajae Malek : *Études et Simulations Numériques de quelques problèmes présentant ou non des singularités.* Thèse soutenue le 28 mai 2022 à la faculté des Sciences, Université Moulay Ismail, Meknès (Maroc).

[2] El Haj Laamri : *Global existence of classical solutions for a class of reaction-diffusion systems*, Acta Appl. Math. 115 (2011), no. 2, 153-165.

[3] El Haj Laamri, Benoît Perthame : *Reaction-diffusion systems with initial data of low regularity.* J. Differential Equations 269 (2020), no. 11, 9310–9335.

[4] El Haj Laamri, Michel Pierre : *Global existence for reaction-diffusion systems with nonlinear diffusion and control of mass.* Ann. Inst. H. Poincaré-Anal. Non Linéaire **34** (2017), 571-591.

[5] El Haj Laamri, Michel Pierre : *Stationary reaction-diffusion systems in L^1.* Math. Models Methods Appl. Sci. 28 (2018), no. 11, 2161–2190.

[6] El Haj Laamri and Michel Pierre : *Stationary reaction-diffusion systems in L^1 revisited.* Discrete Contin. Dyn. Syst. Ser. S 14 (2021), no. 2, 455–464.

[7] Michel Pierre, *Global Existence in Reaction-Diffusion Systems with Dissipation of Mass : a Survey.* Milan J. Math. **78**, no. 2 (2010), 417-455.

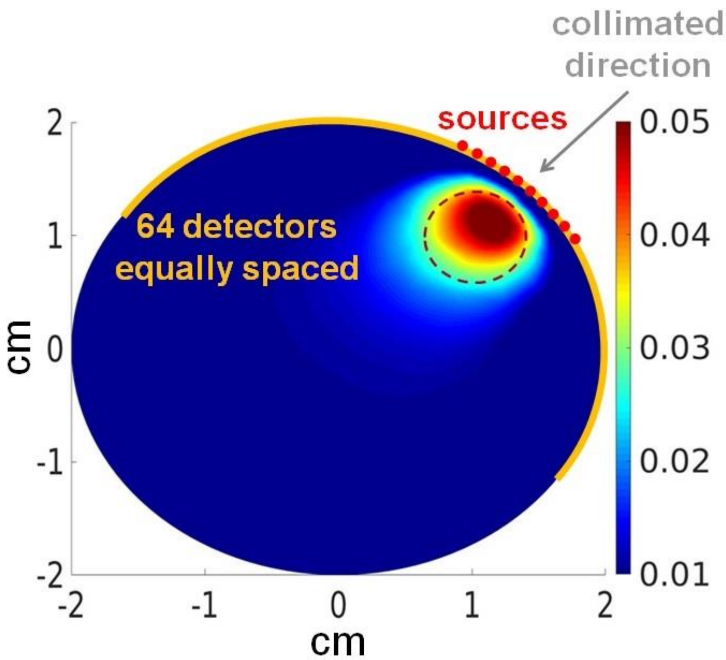
Transfert radiatif dans les milieux semi-transparents avec quelques applications

Fatmir Asllanaj, LEMTA, Nancy

Le transfert radiatif est un mode de transfert de chaleur significatif dans les milieux semi-transparents qui apparait dans de nombreux problèmes d'ingénieries. Il est souvent couplé à la conduction et/ou à la convection. Il peut également être couplé à d'autres phénomènes physiques tels que, la turbulence et la cinétique chimique pour des problèmes de combustion par exemple. Le transfert radiatif dans les milieux semi-transparents à l'échelle mésoscopique, est régi par l'Équation de Transfert Radiatif (ETR). C'est une équation aux dérivées partielles avec un terme intégral par rapport aux deux variables donnant la direction de propagation Ω et contenant des dérivées par rapport au temps t, aux variables donnant la position dans l'espace r et, de direction de propagation lorsque l'on considère un milieu à gradient d'indice. Une variable de longueur d'onde λ s'ajoute lorsque le milieu est non-gris. Le rayonnement dépend ainsi de sept variables au total. Cette dimensionnalité rend difficile la résolution générale de l'ETR tant d'un point de vue théorique que numérique.

Lors de mon exposé, je présenterai de façon synthétique les quelques problèmes de transferts radiatifs que nous avons étudiés avec Jean Rodolphe Roche, professeur à l'IECL. Nous avons abordé ces problèmes avec des études théoriques mathématiques, de nouvelles méthodes numériques de résolution de l'ETR et des problèmes inverses en transfert radiatif et la simulation numérique. Les applications que nous avons étudiées concernent :

- l'amélioration des performances énergétique d'un isolant thermique ;
- l'imagerie optique de fluorescence pour le diagnostic précoce de tumeurs cancéreuses ;
- le rayonnement thermique dans des mélanges de gaz de combustion (CO2 et H2O) lié au problème du réchauffement climatique.



Reconstruction 2D de l'Indocyanine Green (marqueur exogène) dans un tissu biologique par imagerie optique de fluorescence

Examples of finite time blow up in mass dissipative reaction-diffusion systems with superquadratic growth.

Didier Schmitt (IECL Nancy)

Abstract: We provide explicit examples of finite time L^∞-blow up for the solutions of 2×2 reaction-diffusion systems for which three main properties hold: positivity is preserved for all time, the total mass is uniformly controlled and the growth of the nonlinear reaction terms is superquadratic. They are obtained by choosing the space dimension large enough. This is to be compared with recent global existence results of uniformly bounded solutions for the same kind of systems with quadratic or even slightly superquadratic growth depending on the dimension.