Fractional Elliptic Systems with Gradient Source Terms : Numerical Studies and Simulations

Maha DAOUD

Abstract.

The fractional Laplacians $(-\Delta)^s$ of order $0 < s < 1$ constitute the simplest models of nonlocal elliptic operators (see [1] and the references therein). In this talk, we focus on a class of fractional elliptic systems with gradient source terms (see [2]) of the form

$$(S) \qquad \begin{cases} (-\Delta)^s u(\mathbf{x}) &= \|\nabla v(\mathbf{x})\|^q + \lambda f(\mathbf{x}), \quad \mathbf{x} \in \Omega, \\ (-\Delta)^s v(\mathbf{x}) &= \|\nabla u(\mathbf{x})\|^p + \mu g(\mathbf{x}), \quad \mathbf{x} \in \Omega, \\ u(\mathbf{x}) &= v(\mathbf{x}) = 0, \qquad\qquad \mathbf{x} \in \mathbb{R}^N \setminus \Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded regular domain, $N > 2s$ with $\frac{1}{2} < s < 1$, $p, q > 1$, $\lambda, \mu > 0$, and f and g are measurable nonnegative functions. This class of systems arises in a variety of applications such as Fluid dynamics and Engineering (see, for instance, [3]).

In the first part of this presentation, we briefly present some existence and nonexistence results under natural conditions on the data λ, μ, f and g. Moreover, we discuss many open questions.
In the second part, we expose some numerical studies of some open questions. In fact, our simulations confirm the theoretical results and give rise to interesting conjectures. Furthermore, we numerically illustrate the relation between the fractional systems and the classical ones treated in [4].

References

[1] **M. Daoud** ; E.-H. Laamri, *Fractional Laplacians : a short survey.* Discrete and Continuous Dynamical Systems - Series S, 15(1) (2022), 95-116.

[2] S. Atmani ; K. Biroud ; **M. Daoud** ; E.-H. Laamri, *On some nonlocal elliptic systems with gradient source terms.* To appear in Acta Applicandae Mathematicae.

[3] S. Clain, J. Rappaz, M. Swierkosz, R. Touzani, *Numeical modeling of induction heating for two-dimensional geometries.* Mathematical Models and Methods in Applied Sciences 3(6) (1993), 805-822.

[4] B. Abdellaoui, A. Attar ; E.-H. Laamri, *On the existence of positive solutions to semilinear elliptic systems involving gradient term.* Applicable Analysis 98(7) (2019), 1289-1306.