Global existence for a class of reaction-diffusion systems : a numerical study

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Ecology with Lotka-Volterra systems, chemistry with reaction-rate equations, multi-species diffusion of molecules and many other scientific fields lead to reaction-diffusion systems characterized by different diffusion coefficients and satisfying the two natural following properties :
— positivity of the solutions is preserved for all time ;
— the total mass of the components is controlled for all time.

In this presentation, we focus on reaction-diffusion systems modeling reversible chemical reactions. Such systems are of the form

$$(RDS) \begin{cases} 1 \leq i \leq m \\ \partial_t u_i - d_i \Delta u_i & = & f_i(u_1, u_2, \cdots, u_m) & \text{in} & (0, T) \times \Omega \\ \partial_\nu u_i(t, \boldsymbol{x}) & = & 0 & \text{on} & (0, T) \times \partial\Omega \\ u_i(0, \boldsymbol{x}) & = & u_{0,i}(\boldsymbol{x}) \geq 0 & \text{in} & \Omega \end{cases}$$

where for all $i \in \{1, \cdots, m\}$, $d_i > 0$ and $f_i(u_1, \cdots, u_m) = (p_i - q_i) \left(\prod_{j=1}^{m} u_j^{q_j} - \prod_{j=1}^{m} u_j^{p_j} \right)$.

In this talk, we will first recall the known global existence results and the open questions. Then, we will present a numerical study of some open questions. As we will see, our simulations confirm the known theoretical results and give rise to interesting conjecture.

This is a joint work with El Haj Laamri (Université de Lorraine) and Chérif Ziti (Université Moulay Ismail).

Références

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