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On Forms of Kac-Moody Algebras

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Let K be a field of characteristic 0 and $\overline{K}$ its algebraic closure. We want to look to Kac-Moody algebras for generalizations of the theory of semisimple Lie algebras over K.

Here a Kac-Moody algebra $\mathfrak{g} = \mathfrak{g}(A)$ will be the Lie algebra over $\overline{K}$ defined by the generators and relations associated to a generalized Cartan matrix A (also called Kac-Moody matrix: KMM see §4). More precisely $\mathfrak{g}$ is defined as in [**K**] (except that $\mathbb{C}$ is replaced by $\overline{K}$).

A *K-form* of $\mathfrak{g}$ is a Lie algebra $\mathfrak{g}_K$ over K such that there exists an isomorphism from $\mathfrak{g}$ to $\mathfrak{g}_K \otimes \overline{K}$.

If we replace $\overline{K}$ by K in the definition of $\mathfrak{g}$, we obtain a K-form $\mathfrak{g}_K$ which is called *split*. If $K = \mathbb{R}$ a "*compact*" form of $\mathfrak{g}$ is also defined in [**K**]. Some other forms may be found in the literature; but here we want to make a systematic study of all these forms. Their algebraic structures have in fact some likeness with that of the generalizations of Kac-Moody algebras studied e.g. by Borcherds [**Bo**] or Slodowy [**S**].

1. The different kinds of forms

Let $\mathfrak{g}_K$ be a K-form of a Kac-Moody algebra $\mathfrak{g}$ and let us fix an isomorphism from $\mathfrak{g}$ to $\mathfrak{g}_K \otimes \overline{K}$. Then the Galois group $\Gamma = \mathrm{Gal}(\overline{K}/K)$ acts on $\mathfrak{g}$ and $\mathfrak{g}_K$ is identified with the fixed points set $\mathfrak{g}^\Gamma$.

The K-form $\mathfrak{g}_K$ is the direct sum of some indecomposable K-forms. If $\mathfrak{g}_K$ is indecomposable then $\mathfrak{g}_K \otimes \overline{K}$ may not be indecomposable but the indecomposable factors are permuted by Γ and one easily sees that there exists a finite extension K'/K and a K'-form $\mathfrak{g}'_K$ of an indecomposable Kac-Moody algebra $\mathfrak{g}'$ over $\overline{K}$ such that $\mathfrak{g}_K$ may be identified with $\mathfrak{g}'_K$ viewed as a Lie algebra over K.

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So we suppose in the sequel that $\mathfrak{g}$ is *indecomposable and infinite dimensional.*

There is a root space decomposition $\mathfrak{g} = \mathfrak{h} \oplus (\bigoplus_{\alpha\in\Delta} \mathfrak{g}_\alpha)$ with respect to a Cartan subalgebra (CSA) $\mathfrak{h}$, a basis Π of the system $\Delta = \Delta(\mathfrak{g}, \mathfrak{h})$ of roots, a Weyl group W, a set $\Delta_{re} = W\Pi$ of real roots and standard Borel subalgebras $\mathfrak{b}^{\pm} = \mathfrak{h} \oplus (\bigoplus_{\alpha\in\Delta\pm} \mathfrak{g}_\alpha)$.

Let G be the adjoint group of $\mathfrak{g}$ (generated by subgroups U_α for $\alpha \in \Delta_{re}$ acting on $\mathfrak{g}$ as $\exp(\operatorname{ad}\mathfrak{g}_\alpha)$). The CSAs of $\mathfrak{g}$ are conjugated by G [PK], we define N (resp. H) as the stabilizer (resp. fixer) of $\mathfrak{h}$ in G. The Galois group Γ acts on G and we define $G_K = G^\Gamma$.

The Borel subgroup $B^{\pm}$ of G is generated by H and U_α for $\alpha \in \Delta^{\pm}_{re}$. There are two Tits systems in G: (G, B^+, N) and (G, B^-, N). The corresponding parabolic subgroups of G (or the associated parabolic subalgebras of $\mathfrak{g}$) are called respectively positive and negative. Two proper parabolics of different signs are not conjugated by G; moreover for a suitable intrinsic definition of Borel subalgebra [PK] there are exactly two conjugacy classes of Borel subalgebras (or subgroups): those of $\mathfrak{b}^+$ (or B^+) and $\mathfrak{b}^-$ (or B^-).

A (semi-)linear automorphism of $\mathfrak{g}$ acts also on G and the image of a conjugacy class is a conjugacy class; it is said to be *of first kind* if it stabilizes each conjugacy class of Borel subgroup, otherwise it is said to be *of second kind* and it exchanges the two conjugacy classes.

A K-form of $\mathfrak{g}$ is said to be *almost split* if for each γ in Γ the action of γ on $\mathfrak{g}$ is of first kind; otherwise it is said to be *almost anisotropic* (or *almost compact* when $K = \mathbb{R}$).

In fact ([**R4**] using [**KP2**]) the K-form is almost split iff there exists a proper parabolic subalgebra defined over K. So split or quasi split K-forms are almost split (as usual a K-form is *quasi split* if a Borel subalgebra is defined over K). If $K = \mathbb{R}$ the compact form is almost compact.

In [**Héel**] Jean-Yves Hée has constructed the quasi-split Kac-Moody groups over K and some other groups twisted unalgebraically following R. Steinberg and R. Ree's method for Chevalley groups.

2. K-forms of affine algebras

Let $\mathring{\mathfrak{g}}$ be a simple finite dimensional Lie algebra over $\overline{K}$, θ an automorphism of $\mathring{\mathfrak{g}}$ of finite order p and $\zeta \in \overline{K}$ a primitive pth root of unity. For γ in Γ we define $n(\gamma) \in \mathbb{Z}$ (modulo p) such that $\gamma(\zeta) = \zeta^{n(\gamma)}$.

If $\mathring{\mathfrak{g}}_j$ is the eigenspace of θ in $\mathfrak{g}$ corresponding to the eigenvalue ζ^j, $j \in \mathbb{Z}$, one knows that $\mathfrak{g} = (\bigoplus_{j\in\mathbb{Z}} t^j \mathring{\mathfrak{g}}_j) \oplus \overline{K}D \oplus \overline{K}c$ may be endowed with a Lie algebra structure such that $\overline{K}c$ is the center, D acts as $t \cdot \partial/\partial t$ and $[t^j X, t^k Y] = t^{j+k}[X, Y]$ modulo c. In fact any affine Kac-Moody algebra is built like this.

Take now a K-form $\mathring{\mathfrak{g}}_K$ of $\mathring{\mathfrak{g}}$ and consider the action of Γ on $\mathring{\mathfrak{g}}$. Suppose that

There exists an homomorphism $\varepsilon: \Gamma \to \{\pm 1\}$ such that $\gamma\theta\gamma^{-1} = \theta^{\varepsilon(\gamma)n(\gamma)} \forall \gamma \in \Gamma$. Then $\gamma(\mathring{\mathfrak{g}}_j) = \mathring{\mathfrak{g}}_{\varepsilon(\gamma)j}$ and we may define an action of Γ on $\mathfrak{g}$ by

$$\gamma(t^j X) = t^{\varepsilon(\gamma)j}\gamma(X) \qquad \gamma(D) = \varepsilon(\gamma)D \qquad \gamma(c) = \varepsilon(\gamma)c.$$

Then $\mathfrak{g}_K = \mathfrak{g}^\Gamma$ is a K-form of $\mathfrak{g}$ (already defined by Goodman and Wallach **[GW]** when $\theta = \mathrm{Id}$).

This K-form is almost split iff $\varepsilon(\gamma) = 1$ for all γ in Γ.

In fact any almost split form of an affine algebra is as built here. This is proved in **[R3]** when $K = \mathbb{R}$ and in **[B$_3$R]** by Valérie Back for general K. For almost compact real forms the same result is also true, but only if we allow $\mathring{\mathfrak{g}}$ to be semisimple (Ben Messaoud's thesis).

In **[A3]** N. Andruskiewitsch constructs K-forms of $\mathfrak{g}$ using a K-form $\mathring{\mathfrak{g}}_K$ of $\mathring{\mathfrak{g}}$ and a K-form of the associative algebra $K[t, t^{-1}]$.

3. Real-forms of symmetrizable algebras

A real form $\mathfrak{g}_\mathbb{R}$ of $\mathfrak{g}$ corresponds to a conjugate linear involution σ' of $\mathfrak{g}$. Hence the compact form corresponds to a *compact involution* ω'.

If $\mathfrak{g}$ is symmetrizable there exists an invariant nondegenerate bilinear form $B(x, y)$ on $\mathfrak{g}$. Then $H(x, y) = -B(x, \omega' y)$ is hermitian and in fact positive definite on $\bigoplus_{\alpha\in\Delta} \mathfrak{g}_\alpha$. So, after having supposed that σ' and ω' stabilize the same CSA $\mathfrak{h}$, one may generalize a classical proof (see **[H**, III 7.1]) to obtain that

By conjugating by G one may suppose that ω' commutes with σ' **[KP2]** or **[R2]**. Thus $\sigma = \sigma'\omega'$ is a (linear) involution and $\mathfrak{k} = (\mathfrak{g}_\mathbb{R})^\sigma = (\mathfrak{g}_\mathbb{R})^{\omega'}$ is called a *maximal compact subalgebra* of $\mathfrak{g}_\mathbb{R}$.

If $\mathfrak{g}_\mathbb{R}$ is almost compact then σ is of first kind. And one obtains thus **[R2]** a one-to-one correspondence between the conjugacy classes (under G) of (linear) involutions of first kind of $\mathfrak{g}$ and the conjugacy classes (under G) of pairs of an almost compact real form $\mathfrak{g}_\mathbb{R}$ of $\mathfrak{g}$ and a maximal compact subalgebra of it. These compact subalgebras could perhaps be not conjugated under $G_\mathbb{R}$ as the classical proof used above is now available only if σ' and ω' stabilize the same CSA; and this is not always true even if σ' is a compact involution as the Cartan decomposition is false: $G \neq G^{\omega'} H G^{\omega'}$. The compact form is unique up to conjugacy, it corresponds to $\sigma = \mathrm{Id}$.

The (linear) involutions of first kind are classified by Levstein **[L]** (see some corrections and more precisions in **[BR]**).

If $\mathfrak{g}_\mathbb{R}$ is almost split then σ is of second kind. Hechmi Ben Messaoud **[B$_3$R]** has proved (using **[KW]** and §4) that one obtains thus a one-to-one correspondence between the conjugacy classes (under G) of (linear) involutions of second kind of $\mathfrak{g}$ and the conjugacy classes (under G) of almost split real forms of $\mathfrak{g}$. A classification of these forms is also given.

Definitions by generators and relations of these real forms are given by

Berman and Pianzola **[BeP]** for quasi-split and almost compact forms and by Andruskiewitsch **[A1, A2]** for almost compact forms which are inner (i.e. σ is an inner automorphism).

4. Almost split K-forms

A Borel-Tits theory for these forms is developed in **[R4]**, **[R5]**, and **[B$_3$R]**. The principal tools are the buildings $\mathscr{B}^+$ and $\mathscr{B}^-$ associated to the Tits systems (G, B^+, N) and (G, B^-, N), they are twin buildings (a theory developed by Ronan and Tits, see **[T4]**); in particular there are three Bruhat decompositions: $G = B^+NB^+ = B^-NB^- = B^+NB^-$. For nontwisted affine algebras they are Bruhat-Tits buildings of simple algebraïc groups over the valuated fields $\overline{K}((t))$ and $\overline{K}((t^{-1}))$ respectively.

The principal result of **[R4]**, **[R5]** is that for each $\varepsilon = +$ or $-$, G_K is transitive on pairs $(\mathfrak{t}_K, \mathfrak{p}_K^\varepsilon)$ such that $\mathfrak{t}_K \subset \mathfrak{p}_K^\varepsilon$ where $\mathfrak{p}_K^\varepsilon$ is a minimal parabolic subalgebra over K of sign ε and $\mathfrak{t}_K$ a maximal K-split toral subalgebra of $\mathfrak{g}_K$ (i.e. $\mathrm{ad}(\mathfrak{t}_K)$ is diagonalizable in $\mathfrak{g}_K$ and $\mathfrak{t}_K$ is maximal for this property). Moreover the derived algebra $\mathfrak{l}_K$ of the centralizer $\mathfrak{z}_K$ of $\mathfrak{t}_K$ (which is the Levi subalgebra of $\mathfrak{p}_K^\varepsilon$) is a finite dimensional semisimple subalgebra called *K-anisotropic kernel.* There exists a CSA $\mathfrak{h}$ of $\mathfrak{g}$ defined over K such that $\mathfrak{t}_K \subset \mathfrak{h} \subset \mathfrak{p}_K^\varepsilon$.

If P^ε is the parabolic subgroup of G corresponding to $\mathfrak{p}_K^\varepsilon \otimes \overline{K}$, $P_K^\varepsilon = (P^\varepsilon)^\Gamma$ and N' (resp. H') is the stabilizer (resp. fixer) of $\mathfrak{t}_K$ in G_K then $(G_K, P_K^\varepsilon, N')$ is a Tits system; its building is $\mathscr{B}_K^\varepsilon = (\mathscr{B}^\varepsilon)^\Gamma$.

As in the classical case **[T1]** one may define a $*$-action of Γ on Π such that if $\mathfrak{p}$ is a parabolic subalgebra of type $X \subset \Pi$ then $\gamma(\mathfrak{p})$ is of type $\gamma^*(X)$. In fact if $\mathfrak{b}$ is a Borel subalgebra such that $\mathfrak{h} \subset \mathfrak{b} \subset \mathfrak{p}_K^\varepsilon$ then $\gamma(\mathfrak{b})$ is another such Borel subalgebra; so there exists w in the Weyl group of $(\mathfrak{z}, \mathfrak{h})$ such that $w\gamma(\mathfrak{b}) = \mathfrak{b}$. The action γ^* on Π is induced by $w\gamma$ and so is compatible with the Dynkin diagram.

The *index* of $\mathfrak{g}_K$ is the data consisting of the Dynkin diagram, the $*$-action of Γ on it and $\Pi_0 = \mathrm{type}(\mathfrak{p}_K^+) \subset \Pi$. As in the classical case the knowledge of the anisotropic kernel $\mathfrak{l}_K$ and of the index determine the K-form $\mathfrak{g}_K$. This form is quasi split iff $\Pi_0 = \emptyset$ and split iff moreover the $*$-action is trivial. The problem of telling whether such a pair comes actually from a K-form may be reduced to rank one (when $\Pi - \Pi_0$ is a single orbit of the $*$-action) **[B$_3$R]**.

The *relative root system* $\Delta' = \Delta'(\mathfrak{g}_K, \mathfrak{t}_K) = \{\alpha' = \alpha_{|\mathfrak{t}_K} \neq 0/\alpha \in \Delta(\mathfrak{g}, \mathfrak{h})\}$ is more surprising (work of Nicole Bardy, **[B$_3$R]**). It has a basis $\Pi' = \{\alpha' \neq 0/\alpha \in \Pi\} = \{\alpha_i'/i \in J\}$ indexed by the set J of orbits of the $*$-action on $\Pi - \Pi_0$ (for $\alpha, \beta \in \Pi$, $\alpha' = \beta' \neq 0$ iff α and β are in the same orbit in $\Pi - \Pi_0$). The problem is that, as in Borcherd's work **[Bo]**, an element of the basis may be imaginary (in the sense that no reflection correspond to this root): α_i' is imaginary when $\prod_0 \cup \Gamma^*\alpha_i$ is not of finite type. In our case all positive integer multiples of an imaginary root are still roots (see **[R5]** and **[B$_3$R]**).

One may define coroots $(\alpha_i^*)_{i\in J}$ in $\mathfrak{t}_K$: then $A' = (a_{i,j})_{i,j\in J}$ where $a_{i,j} = \alpha_j'(\alpha_i^*)$ is a *relative Kac-Moody matrix* (RKMM instead of generalized generalized Cartan matrix) in the sense that

$$a_{i,j} \in \mathbb{Z}; \qquad a_{i,i} \leq 2; \qquad a_{i,j} \leq 0 \quad \text{for } i \neq j; \qquad a_{i,j} = 0 \Leftrightarrow a_{j,i} = 0.$$

The relative Weyl group $W' = N'/H'$ is simply transitive on the set of minimal parabolic subalgebras over K of sign ε and containing $\mathfrak{t}$; it is generated by reflections r_i for $i \in J$ such that $a_{i,i} > 0$ (i.e. such that α_i' is a real simple root). These reflections are defined by $r_i(h) = h-(2\alpha_i'(h)/a_{i,i})\alpha_i^*$; this says that α_i^* is the coroot of $(2/a_{i,i})\alpha_i'$.

The relative root system is the only subset Δ' of $\bigoplus_{i\in J} \mathbb{Z}\alpha_i'$ such that

(1) $$\Delta' = \Delta_+' \cup \Delta_-' \quad \text{where } -\Delta_-' = \Delta_+' = \Delta' \cap \left(\bigoplus_{i\in J} \mathbb{N}\alpha_i'\right)$$

(2) $$\begin{aligned} \mathbb{N}\alpha_i' \cap \Delta_+' &= \{\alpha_i'\} \quad \text{if } a_{i,i} = 2, \\ &= \{\alpha_i', 2\alpha_i'\} \quad \text{if } a_{i,i} = 1, \\ &= \mathbb{N}^*\alpha_i' \quad \text{if } a_{i,i} \leq 0, \end{aligned}$$

(3) $$\forall \alpha' \in \Delta_+' - \Pi' \quad \exists i \in J \text{ such that } \alpha' - \alpha_i' \in \Delta_+',$$

(4) $$\forall i \in J,\ \forall \alpha' \in \Delta_+' \backslash \mathbb{N}\alpha_i' \text{ then } \Delta' \cap (\alpha' + \mathbb{Z}\alpha_i') \text{ is equal to:}$$

the string $\{\alpha' - p\alpha_i', \dots, \alpha' + q\alpha_i'\}$ with $p, q \in \mathbb{N}$ such that $p - q = (2/a_{i,i})\alpha'(\alpha_i^*)$ if $a_{i,i} > 0$,
$\{\alpha'\}$ if $a_{i,i} \leq 0$ and $\mathrm{supp}(\alpha')$ and $\{\alpha_i'\}$ are not linked (with respect to A'),
a set containing $\alpha' + \mathbb{N}\alpha_i'$ if $a_{i,i} \leq 0$ and $\mathrm{supp}(\alpha')$ and $\{\alpha_i'\}$ are linked.

This generalizes a well-known result for Kac-Moody matrices **[K**, Example 3.5].

To each RKMM an abstract root system with these four properties may be associated (N. Bardy). The classical root system BC_n is associated to a *relative Cartan matrix* (i.e. an RKMM such that $a_{i,i} > 0$ for all i and if $b_{i,j} = (2/a_{i,i})a_{i,j}$ then $(b_{i,j})$ is a Cartan matrix).

The root systems built like this have some good properties (with respect to quotients by groups of permutations of the basis (cf. **[Hée2]**) or by some subspaces or with respect to subsystems) but as shown by Moody and Pianzola **[MP]** it is necessary to consider a generalization where Π' is not free and not finite.

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